Scattering and Transmission Matrix Representations of Multiguide Junctions

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Abstract—**The scattering and transmission matrix representations of the mode-matching technique are generalized for multiguide junctions with arbitrarily shaped coupling apertures. A comparison between both representations is given with respect to the CPU time. The relative convergence phenomenon arising in cascaded discontinuities of multiguide junctions is investigated. A numerical criterion for choosing the correct mode ratio among the guides is presented. Also studied is the application to a kind of multiguide junction between a circular waveguide and a coaxial waveguide with hollow inner.**

I. Introduction

MULTIGUIDE JUNCTIONS are frequently encountered in filters, couplers, slow-wave structures and so on. Electromagnetic scattering problem at waveguide junctions has been investigated by means of various techniques. The most general and rigorous one is the Mode-Matching Technique (MMT), which has received great attention of numerous researchers since several decades ago [1]–[7]. This technique employs efficient field expansions in each guide considered and provides a formally exact solution with matrices of infinite size. Nevertheless, in application of the technique the truncation problem of numerical solutions and the proper choice of the eigenmode ratio among the guides are worthy to be carefully considered.

It is a widely accepted view that as long as the number of modes used in each guide is large enough, the choice of mode ratio does not affect the numerical results significantly. It has been shown, however, in [3] that for some waveguide discontinuities, the requirement of an equal number of modes at both sides of an abrupt junction may violate the boundary conditions, resulting in incorrect numerical solutions, which was called the relative convergence phenomenon and was mathematically analysed in [8] and [9]. In this paper, we will discuss the relative convergence phenomenon arising in the cascaded discontinuities of multiguide junctions. The numerical results reveal that as the septum between any two junctions gets thin enough, the boundary conditions can be exactly satisfied by choosing only a unique mode ratio. With the other mode ratios, the solutions converge to some different values from the correct result. A numerical criterion for choosing the optimal mode ratio is suggested, which should be proportional to the ratio of the waveguide dimensions. Moreover it will be shown that the relative convergence problem can be eliminated or alleviated by using the optimal mode ratio or with a incident mode of higher frequency.

Choosing an equal number of modes at both sides of an abrupt junction or in each guide considered, which is required by a transmission matrix representation (TMR), deviates normally from the optimal mode ratio. It means that the TMR can not be applied without error as long as an infinitely thin septum between two junctions is considered. On the other hand, even if the TMR has a simpler form of matrix calculations than that of a scattering matrix representation (SMR), we will point out that concerning the CPU time the TMR is not definitely superior to the SMR, because the matrix calculations absorb just a minor part of the CPU time in the whole computation. In fact, using an equal number of modes in each guide does not significantly contribute to the convergence, but increase the CPU time. These disadvantages, however, do not prevent the TMR being applied to the problem of cascaded discontinuities with finitely thick septa among the junctions provided the CPU time is immaterial to the computation, because the overall transmission matrix can be obtained much simplier than the overall scattering matrix.

Multiguide discontinuities for an N-furcated parallel-plate junction were studied in [3] and [4]. Another special case of a multiple-aperture junction with an infinitely thin conducting sheet lying in the abrupt plane was investigated in [5]. In [6] and [7] the multiguide discontinuities were regarded as the two-port discontinuity and some interesting generalizations of the SMR were obtained. In this paper, we will generalize the solutions of two-port junction derived in [2] and deduce some concise and generalized formulations for both the SMR and the TMR, which could be applicable to the multiguide junctions with arbitrarily shaped coupling apertures. The formulations will also be generalized for the cascaded discontinuities.

Furthermore, a kind of multiguide junction between a circular waveguide and a coaxial waveguide with a hollow inner, which is typically used in the Marcatili Coupler [10] and the coupled-cavity slow-wave structure of a travelling-wave tube (TWT) [11], will be employed to illustrate the applicability of the technique. In the devel-

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opment of computer models of the TWT, it is required to obtain information about the phase change and the dispersion property of the coupled-cavity. Usually this information is acquired from the expensive "cold test" measurement. Direct calculation is difficult because of the complexity of the shapes involved [11]. It is therefore desirable to develope a generalized theory to get the information from effective field theoretical simulations of CAD techniques.

II. Generalized Formulations for Multiguide Junctions

A. Continuity of Transverse Electric and Magnetic Fields

A typical multiguide junction under consideration, the so called k-furcated guide discontinuity, is illustrated in Fig. 1, which contains one guide with the largest coupling aperture of any shape at one side and k small guides with arbitrarily shaped apertures at another side. The other types of multiguide junctions are essentially equivalent to this type. The transverse electric and magnetic fields can be expanded in terms of the eigenmodes in each guide adjacent to the junction as

$$E_t^{(i)} = [e^{(i)}] \cdot ([\lambda^{(i)}] \cdot [a^{(i)}] + [\lambda^{(i)}]^{-1} \cdot [b^{(i)}]) \quad \text{(1a)}$$

$$H_t^{(i)} = [h^{(i)}] \cdot ([\lambda^{(i)}] \cdot [a^{(i)}] - [\lambda^{(i)}]^{-1} \cdot [b^{(i)}]) \quad \text{(1b)}$$

where

- $[a^{(i)}]$ is an $(M^{(i)} \times 1)$ column matrix with elements $a_m^{(i)}$,
- $[b^{(i)}]$ is an $(M^{(i)} \times 1)$ column matrix with elements $b_m^{(i)}$,
- $[\lambda^{(i)}]$ is an $(M^{(i)} \times M^{(i)})$ diagonal matrix with elements $\exp(-\gamma_m^{(i)} \cdot z)$
- $[e^{(i)}]$ is an $(1 \times M^{(i)})$ row matrix with elements $e_m^{(i)}$,
- $[h^{(i)}]$ is an $(1 \times M^{(i)})$ row matrix with elements $h_m^{(i)}$,

and the superscripts "i" denote the ith guide, i = I, 1, 2, $\cdots$, k; M means the number of expansion modes, $m = 1, 2, \cdots, M$; a_m and b_m are the complex amplitudes of the mth incident mode and scattered mode, respectively; $\gamma_m = \alpha_m + j\beta_m$ means propagation constant of the mth mode; e_m and h_m are the transverse field components of the mth mode, which satisfy the following orthogonality relation:

$$\int_{S_i} (e_m^{(i)} \times h_n^{(i)*}) \cdot dS = Q_n^{(i)} \cdot \delta_{nm},$$

$$i = I, 1, 2, \cdots, k. \quad (2)$$

Now we introduce some definitions

$$[a^{(II)}] = \begin{bmatrix} [a^{(1)}] \\ [a^{(2)}] \\ \vdots \\ [a^{(k)}] \end{bmatrix}, \quad [b^{(II)}] = \begin{bmatrix} [b^{(1)}] \\ [b^{(2)}] \\ \vdots \\ [b^{(k)}] \end{bmatrix},$$

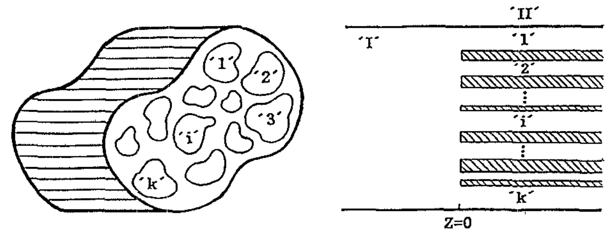

Fig. 1. A typical multiguide junction with k-furcated waveguides.

$$[Q^{(II)}] = \begin{bmatrix} [Q^{(1)}] & & & \\ & [Q^{(2)}] & & \\ & & \ddots & \\ & & & [Q^{(k)}] \end{bmatrix},$$

and

$$[A] = [[A^{(I1)}], [A^{(I2)}], \cdots, [A^{(Ik)}]], \quad (3)$$

where $[Q^{(i)}]$ is an $(M^{(i)} \times M^{(i)})$ diagonal matrix with elements $Qn^{(i)}$ and $[A^{(Ii)}]$ is an $(M^{(I)} \times M^{(i)})$ matrix between guide "I" and "i" with elements

$$A_{nm}^{(Ii)} = \int_{S_i} (e_m^{(i)} \times h_n^{(I)*}) \cdot dS, \quad i = 1, 2, \cdots, k. \quad (4)$$

Furthermore, we define two auxiliary matrices:

$$[R] = [Q^{(I)}]^{-1} \cdot [A], \quad \text{an } (M^{(I)} \times M^{(II)}) \text{ matrix}, \quad \text{(5a)}$$

$$[T] = [[Q^{(II)}]^{-1} \cdot [A]^t]^*, \quad \text{an } (M^{(II)} \times M^{(I)}) \text{ matrix}, \quad \text{(5b)}$$

where $M^{(II)} = \Sigma_{i=1}^k M^{(i)}$ and $[A]^{t*}$ means the transpose conjugate matrix of $[A]$.

Considering the boundary conditions required on the junction plane $Z = 0$, we obtain the following matrix equations:

$$[a^{(I)}] + [b^{(I)}] = [R] \cdot ([a^{(II)}] + [b^{(II)}]), \quad \text{(6a)}$$

$$[b^{(II)}] - [a^{(II)}] = [T] \cdot ([a^{(I)}] - [b^{(I)}]). \quad \text{(6b)}$$

B. Scattering Matrix Representation

Considering the multiguide junction as a generalized 2-port discontinuity, the relations among the incident and scattered modes can be expressed by the SMR as

$$\begin{bmatrix} b^{(I)} \\ b^{(II)} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a^{(I)} \\ a^{(II)} \end{bmatrix}. \quad (7)$$

With some algebraic manipulations, the submatrices of the generalized S-matrix can be derived from (6) as

$$[S_{11}] = [I] - [D], \quad \text{an } (M^{(I)} \times M^{(I)}) \text{ matrix},$$

$$[S_{12}] = [D] \cdot [R], \quad \text{an } (M^{(I)} \times M^{(II)}) \text{ matrix},$$

$$[S_{21}] = [T] \cdot [D], \quad \text{an } (M^{(II)} \times M^{(I)}) \text{ matrix,}$$

$$[S_{22}] = [I] - [T] \cdot [S_{12}], \quad \text{an } (M^{(II)} \times M^{(II)}) \text{ matrix,} \tag{8}$$

where $[I]$ is the unity matrix and

$$[D] = 2 \cdot [[I] + [R] \cdot [T]]^{-1}, \quad \text{an } (M^{(I)} \times M^{(I)}) \text{ matrix.} \tag{9}$$

With (8) and (9), the generalized S-matrix can be rewritten as

$$[S] = \begin{bmatrix} I - D, & D \cdot R^{(I1)}, & D \cdot R^{(I2)}, & \cdots, & D \cdot R^{(Ik)} \\ T^{(I1)} \cdot D, & I - T^{(I1)} \cdot D \cdot R^{(I1)}, & -T^{(I1)} \cdot D \cdot R^{(I2)}, & \cdots, & -T^{(I1)} \cdot D \cdot R^{(Ik)} \\ T^{(I2)} \cdot D, & -T^{(I2)} \cdot D \cdot R^{(I1)}, & I - T^{(I2)} \cdot D \cdot R^{(I2)}, & \cdots, & -T^{(I2)} \cdot D \cdot R^{(Ik)} \\ \vdots & \vdots & \vdots & \cdots, & \vdots \\ T^{(Ik)} \cdot D, & -T^{(Ik)} \cdot D \cdot R^{(I1)}, & -T^{(Ik)} \cdot D \cdot R^{(I2)}, & \cdots, & I - T^{(Ik)} \cdot D \cdot R^{(Ik)} \end{bmatrix} \tag{10}$$

where

$$[R^{(Ii)}] = [Q^{(I)}]^{-1} \cdot [A^{(Ii)}] = [[I] - [S_{11}^{(Ii)}]]^{-1} \cdot [S_{12}^{(Ii)}], \tag{11a}$$

$$[T^{(Ii)}] = [[Q^{(i)}]^{-1} \cdot [A^{(Ii)}]^t]^* = [S_{21}^{(Ii)}] \cdot [[I] - [S_{11}^{(Ii)}]]^{-1}. \tag{11b}$$

Thus, the S-matrix of a multiguide junction is represented by the individual matrix $[R^{(Ii)}]$ and $[T^{(Ii)}]$ or the S-parameter $[S^{(Ii)}]$ of the 2-port discontinuity.

C. Transmission Matrix Representation

The relations among the incident and scattered modes of multiguide discontinuities can also be expressed by the TMR as

$$\begin{bmatrix} a^{(I)} \\ b^{(I)} \end{bmatrix} = \begin{bmatrix} [U] & [V] \\ [V] & [U] \end{bmatrix} \cdot \begin{bmatrix} a^{(II)} \\ b^{(II)} \end{bmatrix}. \tag{12}$$

According to the different choices of the mode ratio, there are two ways to obtain the submatrices of the generalized T-matrix.

(i) If the mode numbers at both sides of the junction are chosen to be the same, namely, $M^{(I)} = M^{(II)}$, $[R]$ and $[T]$ defined in (5) are invertible matrices, so that a formulation similar to the TMR of the 2-port junction can be derived from (6):

$$[U] = ([R] - [T]^{-1})/2$$

and

$$[V] = ([R] + [T]^{-1})/2. \tag{13}$$

For some discontinuities, the presetting of $M^{(I)} = M^{(II)}$ might violate the boundary conditions and lead to some incorrect results, which is presented in [3] and also in Section III of this paper.

(ii) If the mode number in each guide considered is chosen to be the same, namely, $M^{(I)} = M^{(i)}$ ($i = 1, 2, \cdots, k$), $[R^{(Ii)}]$ and $[T^{(Ii)}]$ defined in (11) are invertible matrices. Another form of the T-matrix for a multiguide junction can be derived from (6):

$$[U] = [[U^{(I1)}], [U^{(I2)}], \cdots, [U^{(Ik)}]],$$
$$[V] = [[V^{(I1)}], [V^{(I2)}], \cdots, [V^{(Ik)}]], \tag{14a}$$

with

$$[U^{(Ii)}] = (k \cdot [R^{(Ii)}] - [T^{(Ii)}]^{-1})/2k,$$
$$[V^{(Ii)}] = (k \cdot [R^{(Ii)}] + [T^{(Ii)}]^{-1})/2k,$$
$$i = 1, 2, \cdots, k. \tag{14b}$$

It will be demonstrated in this paper that the requirement of an equal number of modes in each guide may result in an incorrect numerical solution too.

D. Scattering Matrix Representation for Cascaded Discontinuities

We discuss only the cascaded discontinuities of two multiguide junctions separated with a length L, shown in Fig. 2. With the formulations given in [2], one can proceed to get the overall S-matrix of cascaded series. From (7) we have

$$\begin{bmatrix} b^{(I)} \\ b_A^{(II)} \end{bmatrix} = \begin{bmatrix} S_{11}^A & S_{12}^A \\ S_{21}^A & S_{22}^A \end{bmatrix} \cdot \begin{bmatrix} a^{(I)} \\ a_A^{(II)} \end{bmatrix}, \quad \text{for junction } A \tag{15a}$$

and

$$\begin{bmatrix} b^{(III)} \\ b_B^{(II)} \end{bmatrix} = \begin{bmatrix} S_{11}^B & S_{12}^B \\ S_{21}^B & S_{22}^B \end{bmatrix} \cdot \begin{bmatrix} a^{(III)} \\ a_B^{(II)} \end{bmatrix}, \quad \text{for junction } B \tag{15b}$$

In section "II" between junction A and B, the S-matrix $[S^{(II)}]$ can be defined as

$$\begin{bmatrix} a_A^{(II)} \\ a_B^{(II)} \end{bmatrix} = \begin{bmatrix} S_{11}^{(II)} & S_{12}^{(II)} \\ S_{21}^{(II)} & S_{22}^{(II)} \end{bmatrix} \cdot \begin{bmatrix} b_A^{(II)} \\ b_B^{(II)} \end{bmatrix}, \tag{16a}$$

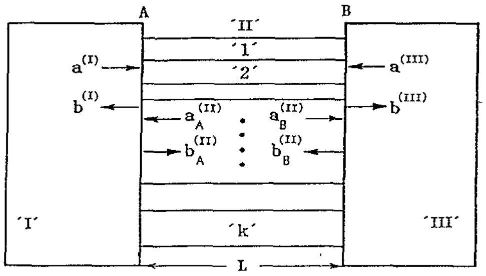

Fig. 2. Multiguide discontinuities with two abrupt junctions.

with

$$[S_{ij}^{(II)}] = \begin{bmatrix} S_{ij}^{(1)} & & & \\ & S_{ij}^{(2)} & & \\ & & \ddots & \\ & & & S_{ij}^{(k)} \end{bmatrix}, \quad i, j = 1, 2. \tag{16b}$$

where $[S^{(i)}]$, $i = 1, 2, \cdots, k$, is the S-matrix from junction A to B through the ith furcated guide, which may consists of some abrupt junctions or simply a uniform line. For the uniform line we have

$$[S^{(i)}] = \begin{bmatrix} [0] & [\lambda^{(i)}] \\ [\lambda^{(i)}] & [0] \end{bmatrix}. \tag{17}$$

With some algebraic computations for (15) and (16), the combined S-matrix from guide "I" to "III" can be found out

$$\begin{bmatrix} b^{(I)} \\ b^{(III)} \end{bmatrix} = \left(\begin{bmatrix} S_{11}^{A} & \\ & S_{11}^{B} \end{bmatrix} + \begin{bmatrix} S_{12}^{A} & \\ & S_{12}^{B} \end{bmatrix} \cdot \left[[I] - [S^{(II)}] \cdot \begin{bmatrix} S_{22}^{A} & \\ & S_{22}^{B} \end{bmatrix} \right]^{-1} \cdot [S^{(II)}] \cdot \begin{bmatrix} S_{21}^{A} & \\ & S_{21}^{B} \end{bmatrix} \right) \cdot \begin{bmatrix} a^{(I)} \\ a^{(III)} \end{bmatrix}. \tag{18}$$

In order to avoid the matrix inversion of $[S^{(II)}]^{-1}$, two more matrix multiplications are employed in (18).

E. Transmission Matrix Representation for Cascaded Discontinuities

In a similar way to obtain $[S^{(II)}]$, the T-matrix for the section "II" can be defined as

$$\begin{bmatrix} b_A^{(II)} \\ a_A^{(II)} \end{bmatrix} = \begin{bmatrix} T_{11}^{(II)} & T_{12}^{(II)} \\ T_{21}^{(II)} & T_{22}^{(II)} \end{bmatrix} \cdot \begin{bmatrix} b^{(II)} \\ a_B^{(II)} \end{bmatrix}, \tag{19a}$$

with

$$[T_{ij}^{(II)}] = \begin{bmatrix} T_{ij}^{(1)} & & & \\ & T_{ij}^{(2)} & & \\ & & \ddots & \\ & & & T_{ij}^{(k)} \end{bmatrix}, \quad i, j = 1, 2. \tag{19b}$$

where $[T^{(i)}]$, $i = 1, 2, \cdots, k$, is the T-matrix through the ith furcated guide. Again for a uniform line we have

$$[T^{(i)}] = \begin{bmatrix} [0] & [\lambda^{(i)}]^{-1} \\ [\lambda^{(i)}] & [0] \end{bmatrix}. \tag{20}$$

With $[T^{(II)}]$, the combined T-matrix from guide "I" to "III" can be simply obtained from

$$\begin{bmatrix} a^{(I)} \\ b^{(I)} \end{bmatrix} = [T^{A}] \cdot \begin{bmatrix} T_{21}^{(II)} & T_{22}^{(II)} \\ T_{11}^{(II)} & T_{12}^{(II)} \end{bmatrix} \cdot [T^{B}]^{-1} \cdot \begin{bmatrix} a^{(III)} \\ b^{(III)} \end{bmatrix}. \tag{21}$$

where $[T^A]$ and $[T^B]$ are the T-matrices of junction A and B, which can be found out from (13) or (14), depending on the choice of eigenmode ratio. It is obvious that computing the combined T-matrix in (21) is much easier than doing the combined S-matrix in (18). Furthermore the overall T-matrix can be achieved by simply multiplicating the individual T-matrices of all cascaded series. From this point of view, the TMR gains an advantage over the SMR. However, it should be noted that due to $[\lambda^{(i)}]^{-1}$ of the evanescent modes, the T-matrix of a uniform line may contain exponential functions with positive arguments, which lead to an overflow in the computation. A combinative application of the TMR and the SMR is therefore recommended for the cascaded discontinuities of multiguide junctions.

III. Application to a Multiguide Junction between a Circular Waveguide and a Coaxial Waveguide with Hollow Inner

To illustrate the applicability of the formulations derived above, we consider a 4-guide discontinuities with two abrupt junctions, shown in Fig. 3, which is typically used in the coupled-cavity slow-wave structure and the Marcatili coupler. In this structure, a kind of multiguide junction between a circular waveguide and a coaxial waveguide with hollow inner is involved. In order to analyse the effect of different mode ratios on the numerical results, the SMR is employed. For the discontinuity with a single junction related to the first three guides, Fig. 4 shows the Er components of transverse electric fields at both sides of the junction, in which both continuity and convergence of the transverse field are presented.

A. Optimal Choice of the Mode Ratio

Although the correct or close to the correct solution can be obtained by choosing different mode ratios, especially for the discontinuity with single junction, there is a unique mode ratio for the best numerical approximation. On the other hand, choosing some mode ratios does not significantly benefits the numerical computation but increase the CPU time, even though they might lead to correct solutions in some cases. It is, hence, important to find out an optimal mode ratio as a guide for choosing the eigenmodes. We found that the cutoff frequencies of higher order modes can serve as a criterion for the correct choice

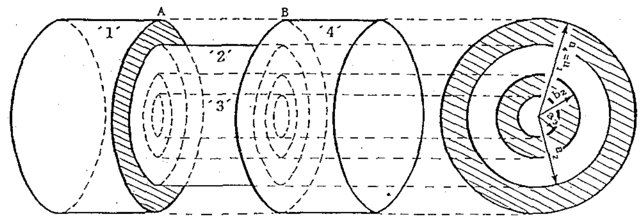

Fig. 3. 4-guide discontinuities with two abrupt junctions. "1": circular waveguide with radius a_1, "2": coaxial waveguide with outer radius a_2 and inner radius b_2, "3": circular waveguide with radius a_3, "4": circular waveguide with radius $a_4 = a_1$.

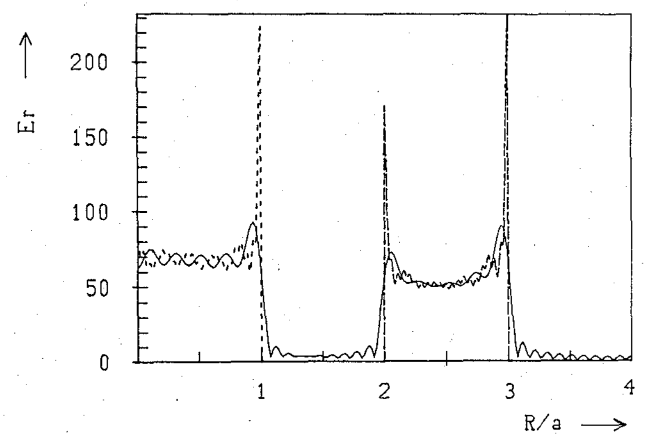

Fig. 4. Transverse electric field along R axis for the 3-guide discontinuity with a single junction. TE_{11} with $\lambda = 5a$ incidence from "1". "1": solid, 40 modes used, $a_1 = 4a$; "2": long dash, 40 modes used, $a_2 = 3a$, $b_2 = 2a$; "3": short dash, 40 modes used, $a_3 = a$.

of the mode ratio. It means that the field expansion series in (1) should be truncated so that the cutoff frequency of the highest order mode in each guide is roughly the same. Hence, the mode ratio, being proportional to the ratio of the guide dimensions, can be considered as the correct choice, by reason that the cutoff frequency is approximately proportional to the ratio of mode number to guide dimension in many cases. Thus we have

$$Mo^{(I)}:Mo^{(1)}:Mo^{(2)}: \cdots :Mo^{(k)}:Mo^{(III)} = d^{(I)}:d^{(1)}:d^{(2)}: \cdots :d^{(k)}:d^{(III)} \quad (22)$$

where $Mo^{(i)}$ is the optimal mode number in the ith guide and $d^{(i)}$ is the dimension of the ith guide. For the structure in Fig. 3, guide "1" and "4" have the same dimensions, so $Mo^{(1)} = Mo^{(4)}$.

B. Relative Convergence Problem

For the discontinuities with two or more abrupt junctions, the effect of evanescent modes with higher order can not be neglected, when the septa between the junctions get thin enough. In that case, the evanescent modes excited at different junctions might strongly interfere with each other. With relatively more modes used in the multiguide block between two junctions, more evanescent modes will be excited. As a result, the interference of evanescent modes become so strong that it ruins the boundary and edge conditions, resulting in uncorrect solutions—the so called relative convergence phenomenon. According to the analysis, it can be predicted that the relative convergence problem can be alleviated or eliminated by using less modes among the junctions or with an incident mode of higher frequency.

In Fig. 5 the relative convergence phenomenon is demonstrated by drawing the transverse electric fields. It can be seen from Fig. 5(a) and (b) that with an equal number of modes in each guide or at both sides of the junction, i.e., $M^{(1)}$ $(= M^{(4)}):M^{(2)}:M^{(3)} = 1:1:1$ or $2:1:1$, the numerical solutions fail to satisfy the boundary conditions at the junction for a septum of $L = 10^{-9}$a. Fig. 5(c) demonstrates that the relative convergence problem can be eliminated with the optimal mode ratio of (4 : 1 ; 1). As the septum gets longer, e.g., $L = a$, the effect of higher order modes becomes so week after the attenuation distances that even with a mode ratio of (1 : 1 : 1), the field plot will not be ruined, as shown in Fig. 5(d). Even so, it is evident that using an equal number of modes in each guide or at both sides of the junction consums much more CPU time than choosing the optimal mode ratio. Frequently the SMR with a proper choice of mode ratio is superior to the TMR, as long as the CPU time is considered. By choosing the equal number of modes, the TMR achieves a simpler form of matrix calculations, yet the matrix size has to be increased, which absorbs more CPU time than operating the SMR with complexity of the form. Moreover it has been found that the most CPU time normally is not consumed by matrix calculations, but by other operations such as integration, solving the eigenmodes and eigenequations, etc. Therefore using as few modes as possible will actually save much more CPU time.

In Fig. 6, the convergence of the S-parameter, by taking the magnitude of the dominant mode reflection coefficient $S_{11}(1, 1)$ as an example, has been studied. Fig. 6(a) shows the relative convergence phenomenon for a septum of $L = 10^{-9}$a. It can be seen that even with large numbers of the eigenmodes, the S-parameter does not converge to the same value with different mode ratios. Comparing with the optimal mode ratio of 4 : 1 : 1, if fewer modes are used between the two junctions, e.g., the ratio of (8 : 1 : 1), the convergent values of $S_{11}(1, 1)$ are not greatly in error. However, for $M^{(2)} > Mo^{(2)}$ and $M^{(3)} > Mo^{(3)}$, e.g., with an equal number of modes at both sides of the junction (2 : 1 : 1) or in each guide (1 : 1 : 1), the results deviate rather greatly from the correct value. Using the same percentage (80%) of the convergent value obtained by the optimal ratio as the ordinate scale but with an incident mode of higher frequency, it is interesting to note, in Fig. 6(b), that the relative convergence problem can be alleviated by increasing the effect of the dominant modes, as predicted above. As the septum gets long enough (e.g., $L = a$), the disappearance of the relative convergence is observed in Fig. 6(c), in which the same percentage (80%) for choosing the ordinate scale was used. In this case, the choice of mode ratio does not influence the convergence of S-parameter significantly.

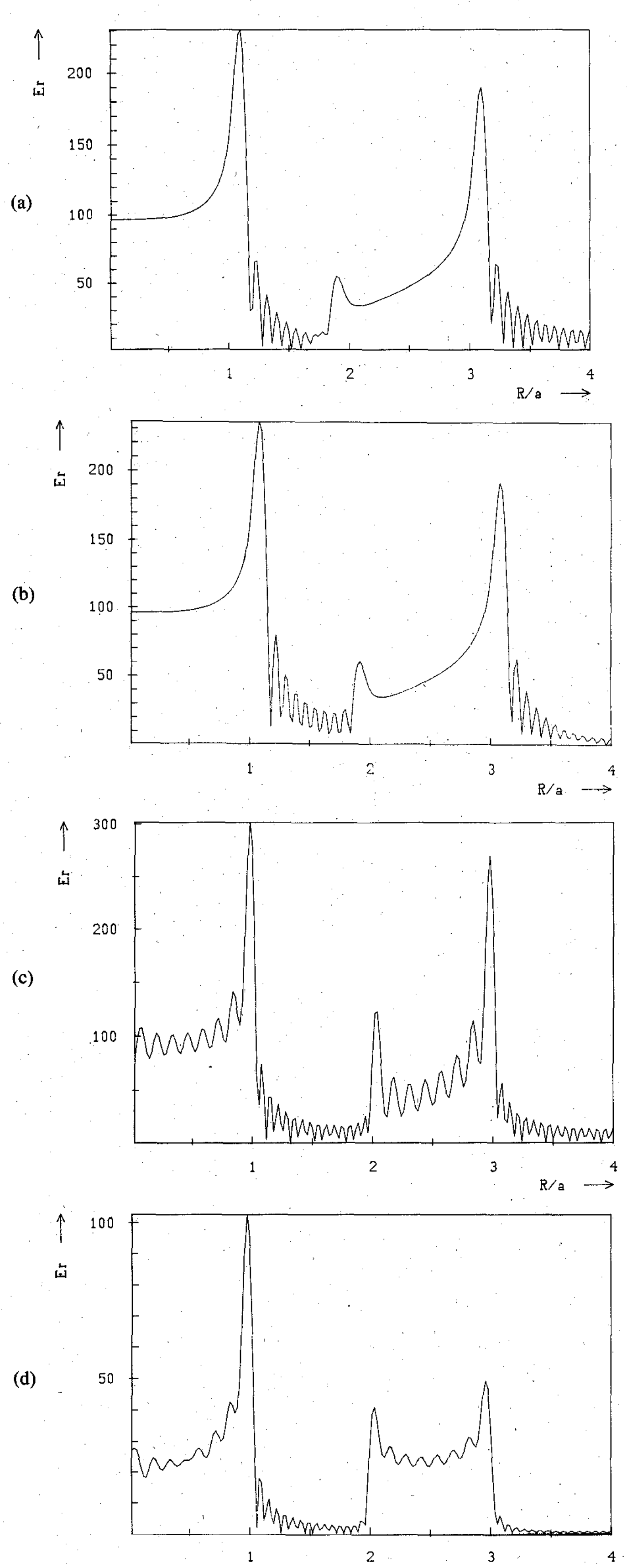

Fig. 5. Relative convergence phenomenon demonstrated by field plots of 4-guide discontinuities with two junctions. TE_{11} with $\lambda = 5a$ incidence from "1", 120 modes used in "1", $a_1 = a_4 = 4a$, $a_2 = 3a$, $b_2 = 2a$, $a_3 = a$. (a) Septum $L = 10^{-9}a$, mode ratio 1 : 1 : 1, CPU time 8 min. (b) Septum $L = 10^{-9}a$, mode ratio 2 : 1 : 1, CPU time 2.2 min. (c) Septum $L = 10^{-9}a$, mode ratio 4 : 1 : 1, CPU time 1.1 min. (d) Septum $L = a$, mode ratio 1 : 1 : 1, CPU time 6.4 min.

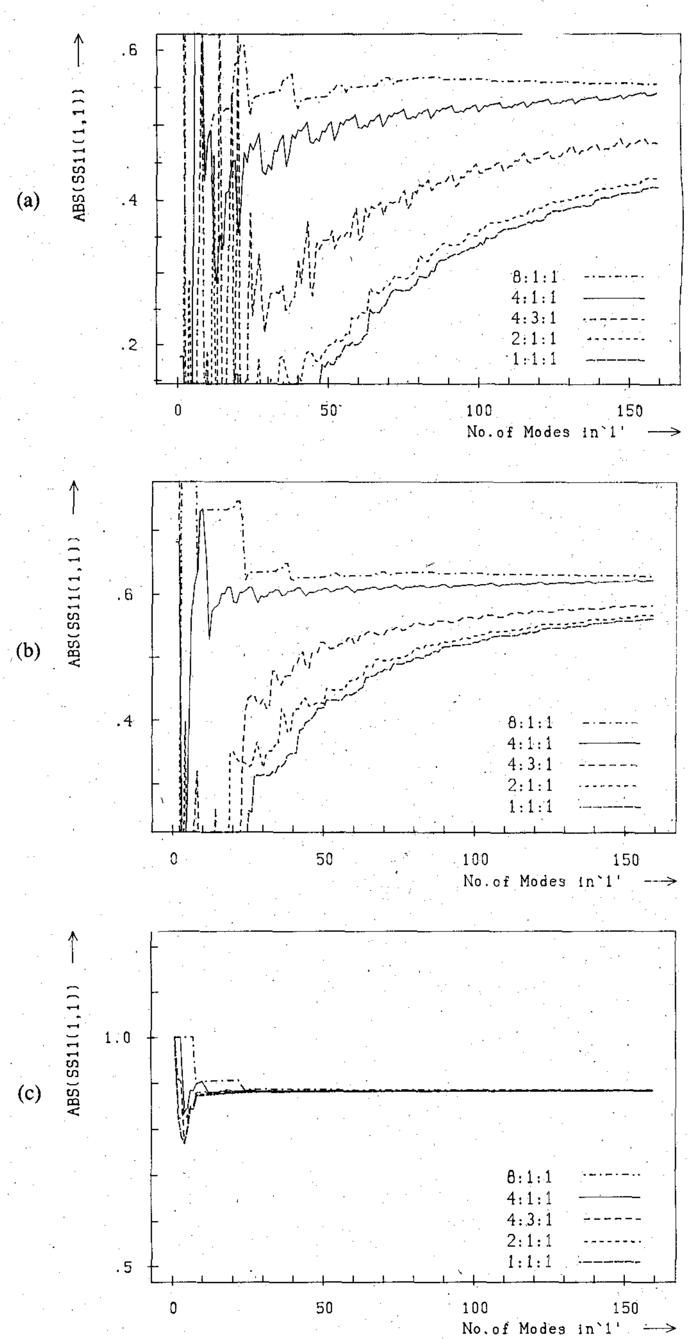

Fig. 6. Relative convergence problem studied by S-parameters of 4-guide discontinuities with two junctions. $a_1 = a_4 = 4a$, $a_2 = 3a$, $b_2 = 2a$, $a_3 = a$. Septum L, incident wavelength λ. (a) $L = 10^{-9}a$, $\lambda = 5a$. (b) $L = 10^{-9}a$, $\lambda = 1.5a$. (c) $L = a$, $\lambda = 5a$.

IV. Conclusion

The scattering and transmission matrix representation of the mode-matching technique have been generalized for multiguide junctions. The prerequisite of this technique is, however, an available solution of the eigenmodes in each guide. The relative convergence phenomenon that occurs in cascaded discontinuities has been analyzed by comparing the solutions obtained under different conditions. It has been shown that even with large number of eigenmodes, the relative convergence might ruin the numerical solutions, as long as the mode ratio is improperly chosen. A numerical criterion for choosing the correct mode ratio has been given, which can be used as a guide to alleviate relative convergence in the computation. The convergence and consistence of the field plots drawn at both sides of the abrupt junction confirm the accuracy of the simulative solutions. Using the formulations obtained above provides a formally exact, fast and rigorous solution to many problems of multiguide junctions, such as filters, couplers, slow-wave structures, finlines and so on.

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References

[1] A. Wexler, "Solution of waveguide discontinuities by modal analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-15, pp. 508–517, Sept. 1967.

[2] A. S. Omar and K. F. Schünemann, "Transmission matrix representation of finline discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 765–770, Sept. 1985.

[3] R. R. Mansour and R. H. Macphie, "An improved transmission matrix formulation of cascaded discontinuities and its application to E-plane circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1490–1498, Dec. 1986.

[4] R. R. Mansour and R. H. Macphie, "Scattering at an N-furcated parallel plate waveguide junction," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 830–835, Sept. 1985.

[5] P. H. Mastermann and P. J. B. Clarricoats, "Computer field-matching solution of waveguide transverse discontinuities," *Proc. Inst. Elec. Eng.*, vol. 118, no. 1, pp. 51–63, Jan. 1971.

[6] J. M. Rebollar and J. A. Encinar, "Field theory analysis of multiport-multidiscontinuity structures: Application to short-circuited E-plane septum," *Proc. Inst. Elec. Eng.*, vol. 135, pt. H, no. 1, pp. 1–7, Feb. 1988.

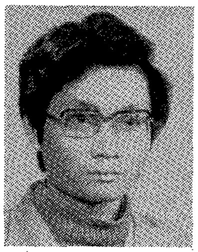
[7] J. Esteban and J. M. Rebollar, "Generalized scattering matrix of generalized two-port discontinuities: Application to four-port and nonsymmetric six-port couplers," *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 1725–1734, Oct. 1991.

[8] S. W. Lee, W. R. Jones and J. J. Campbell, "Convergence of numerical solutions of iris-type discontinuity problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 528–536, June 1971.

[9] R. Mittra, T. Itoh and T. S. Li, "Analytical and numerical studies of the relative convergence phenomenon arising in the solution of an integral equation by the moment method," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 96–104, Feb. 1972.

[10] E. A. Marcatili, "A circular-electric hybrid junction and some channel-dropping filters," *Bell Syst. Tech. J.*, vol. 40, pp. 185–196, 1961.

[11] R. G. Carter and S. Liu, "Method for calculating the properties of coupled-cavity slow-wave structures from their dimensions," *Proc. Inst. Elec. Eng.*, vol. 133, pt. H, no. 5, pp. 330–334, Okt. 1986.



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